Potential to Detect Cosmic Rays ThroughThermo-Acoustic Waves on Lunar Surface by LSGA

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Enjoy your reading!

Abstract

LSGA is a proposed project that will happen on the moon by deploying a two 12 km long optical fiber to detect any movement caused by an acoustic, seismic or gravitational wave. Acoustic waves were not given enough credit in astroparticle physics until the time of ultra-high energy particles that hits a matter and transfer it enough energy to create a sufficient acoustic wave can be detected over several kms. This topic can open a many new doors were not checked before, and might discover new physics. After a study done on this project it will be compared with similar big observatories as Pierre Auger to see what is LSGA flux compared to it.

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1 Introduction

Along with exploring the moon, a project to deploy an optical wire on the lunar surface to gaze up into space for interesting events and sources has been proposed. The Lunar Seismic and Gravitational Antenna (LSGA [1]) is getting ready for an important mission to the moon to detect ultra-high-energy cosmic rays and gravitational waves, as well as examine seismic activity in the moon's crust.

This antenna enables for the detection of ultra-high-energy cosmic rays that collide with the moon and emit acoustic waves. Depending on the quality of transmission of the material of acoustic waves, this technique could allow us to cover a large region of the moon, especially when working with low frequency sounds. It's interesting to investigate this range of cosmic ray spectrum; we frequently discover surprising results that cause us to reconsider our minds, yet we still don't know where they come from.

The shear waves produced by seismic events on the moon can be detected by LSGA. It can enrich geophysics by identifying the geographical and depth distribution of lunar seismic sources, as well as improving knowledge of the type and thickness of the lunar crust and determining the location and state of the core, compared to prior attempts.

Another critical job for LSGA is to detect gravitational waves by using the moon as a spherical detector and detecting the vibrations created by GW, or to detect directly the space-time distortions induced by GW. Through measurements of the Hubble constant and equation of state, weak lensing e ects, in ation and stochastic background, as well as primordial phase transition topological defects and searches for dark matter, this expedition can contribute to several disciplines, namely cosmology. In astronomy, the study of the channels driving the formation and evolution of stellar mass binaries (Binary Neutron Stars . . .) or isolated supernovae bursts and provide measurements of intermediate-mass BHs (IMBHs), including the impact of their evolution environment missing link in the formation and evolution of (super)massive BHs. It is also possible to test fundamental theories such as General Relativity and the Standard Model of Particle Physics.

Since the discovery of Victor Hess of cosmic rays in 1912 and the physics community is in a competition to detect those mysterious particles that the universe throws on us for a better understanding of the universe and its physics. We kept climbing the energy scale where exciting things happen up to discovering ultra-high energy particles by Pierre Auger Observatory and reaching the above GZK limit (5×10^{19} eV) creating a cosmic ray

paradox, it's very di cult to nd an explanation of their existence and very hard to nd a physical process that either generate or accelerate particles at this enormous energy.

One of the goals here is to gure out where they come from, but because of the high energy costs to us that they are very rare, we would have to wait decades to see one. As a result, obtaining additional uxes will require a large detector such as the Pierre Auger Observatory. These high-energy cosmic rays allow us to look on to a previously unexplored part of the universe. Connecting the highest-energy particles with the rest of the universe is a fascinating quest for us because we learn from both sides: how nature produces these particles and how probing these particles reveals new information about the universe's nature.

The LSGA is designed to be sensitive to acoustic waves, which can help identify these ultra-high energy (UHE) CRs via thermo-acoustic waves generated by their showers in the moon's regolith, as detailed by G. A. ASKARIYAN and developed by John G. Learned and V.D.Volovic and G.B.Khristiansen. We may nd the maximum distance we can distinguish the acoustic from the detector noise by solving the pressure or displacement wave di erential equations.

2 Background

2.1 Cosmic-ray energy spectrum

Figure 1: Cosmic-ray energy distribution[4]

summarizing all cosmic ray data in one plot is what called the energy spectrum of cosmic ray ux as function of energy. What we see immediately is that rst the cosmic ray kind of physics spectrum extends over many orders of magnitude in both ux and that it's kind of doing roughly the same thing along the whole spectrum, so this is a log arhythmic plot, it's commonly referred to as a leg, the big features in the gure are called the knee and the ankle, this power law allow us to connect to and interpret the underlying physics of these particles, so the energy spectrum is kind of core to the study of cosmic ray physics.

The shape of this spectrum shows that the vast majority of cosmic rays are at the lowest entities, and the lowest energy cosmic rays are thought to come from energetic processes in the sun, causing them to be accelerated to these somewhat energetic states, and then we have the imaginatively named high energy and very high energy cosmic rays moving up the spectrum, these are thought to come from energetic events in our galaxy, and it's hypothesized that at some point between high and very high energy, perhaps at the knee, the second knee, or the ankle, we should have a transition from galactic to

extragalactic cosmic rays, simply because as the energy of these particles increases, it becomes increasingly di cult to con ne them in small spaces, small spaces here is being the Galaxies.

Ultra-high energy cosmic rays must have been produced in some kind of very violent astrophysical event which is capable of accelerating them, if one wants to learn more about these events, one of the most obvious things to do is look at them through light, which is exactly what people have done. It's also possible to detect matter such as photons that carry information on what's happening in the physics of these sources. As well, high-energy neutrinos, which have very little mass and are not charged, move in straight lines from their sources, which makes them ideal for astronomy because we can just look in the direction they came from.

Other one of the hottest topics about UHECRs is their chemical composition.This area was studied for lower energies, where it was dominant by protons and heavier nuclei like iron(Fe). These studies are veri ed until10^{17} eV using the hadronic(quark) interactions.

It's commonly recognized that there's a signi cant cut-o in the spectrum due to the GZK limit make it be not matched with a straight-line power law. GZK is an upper limit for the energy of cosmic rays coming from far away sources it is named after three physicists Greisen, Zatsepin and Kuzmin who calculated this cut-o energy in 1966 in short this energy limit (5 10^{19} eV) is due to collisions of cosmic rays (protons) with photons from the cosmic microwave background radiation and produce pions which reduces the proton energy.

$$_{CMB} + p \; ! \quad ^{+} \; ! \quad n + \; ^{+} \tag{1}$$

$$_{CMB} + p \; ! \quad ^{+} \; ! \quad p + \; ^{0} \tag{2}$$

2.2 LSGA [1]

2.2.1 The Antenna

It is proposed to install on the lunar surface a narrow-band laser with two ber optic perpendicular arms each of 12 km long with two re ecting mirrors at the edges. Acoustic and temperature sensing ber cables will also be laid in parallel along the same L.

LSGA is based on the method of phase-Optical-Time Domain Re ectometry (

Figure 2: Sketch of the LSGA

ODTR), where acoustic or a seismic wave can be detected by measuring the back-scattered laser light resulted by the displacement of the ber walls caused by the acoustic or seismic wave. Measuring the time it needed the light to travel back, the part of the optical ber that detected the signal can be calculated.

2.2.2 Bene ts of the moon

The moon is a calmer place than the earth, since it has a lower seismic background and has no atmospheric e ects.
The rotation of the moon around the earth allows to localise events by the method of triangulation (similar to parallax, where earth rotates around the sun for one year instead).
The backgrounds' independence from earth or space detectors is a advantage for studies correlated studies for stochastic gravitational energy analyses.

2.3 Pierre Auger observatory and its contributions

The Pierre Auger Observatory is an ultra-high energy cosmic ray observatory in Argentina. It detects the air showers caused by ultra-high energetic cosmic rays in the earth's atmosphere and determine its direction.

It is a grid distribution of water tank detectors that use Cherenkov light to distinguish a particle along with uorescence detectors that are used to track the particle air shower's

glow on cloudless moonless nights, as it descends through the atmosphere. The Auger Observatory has created a 3,000 km^2 detection area.

The initial ndings team behind the Auger Project has released revealed that the 27 highest-energy events' origin directions were linked to the sites of active galactic nuclei (AGNs). Later, using data from 12 years of observations, they managed to discover a large anisotropy of the arrival direction of cosmic rays with energy exceeding 8×10^{18} eV. This supports the idea that these exceptionally high-energy cosmic rays originate from extragalactic sources.

2.4 Theory of Thermo-Acoustic waves

2.4.1 Askariyan Theory

This technique is not used a lot in astronomy and Astroparticle physics. It was rstly introduced by Askariyan In his paper [5]. As a high energy particle enters a medium it makes a shower by interacting with the molecules of the material creating secondary particles, this transfers part of the initial particle to the medium creating a local heating that relaxes in a form of pressure wave.

This wave follows the di erential equation:

$$\frac{\partial^2 P}{\partial t^2} \quad C^2 \quad P = \frac{\partial^2 q}{\partial t^2} \tag{3}$$

q q(t; r) is the deposited energy density distribution and r is the location

represents the Gruneisen parameter and shows the acoustic pulse relative intensity(how good the acoustic pulse travel in the medium)

$$= \frac{C^2}{C_p} \tag{4}$$

and C_p being the thermal expansion coe cient and heat capacity of the medium respectively

The solution to this equation is given by the Kirchho integral assuming that energy is deposited is instantaneously

$$P(t; r) = \frac{C^2}{4 C_p} \frac{\partial}{\partial R} \int d \frac{q(r)}{R} \tag{5}$$

This pressure form is not accounting for the absorption of the medium(will be explained more in detail in next section). To include that, a Fourier transform is done so the pressure wave $\tilde{P}(!; R\ )$ whith $!$ is the angular frequency of the wave.

Near eld

$$\tilde{P}(!; R\ ) = \frac{!}{4\ ^{\frac{3}{2}} C_p} p \frac{E}{RR} \quad \text{for} \quad (R < R\ ) \tag{6}$$

Far eld

$$\tilde{P}(!; R\ ) = \frac{!}{4\ ^{2} C_p} \frac{E}{R} \quad \text{for} \quad (R >> R\ ) \tag{7}$$

Where $R\ = \frac{L^2}{}$ and L is the length of the shower

$= \frac{r_s}{}$ and r_s being the radius of the shower

2.4.2 Learned and Volovic Contribution

Learned starts approaching the problem by solving the wave equation, including the attenuation(Energy loss in the medium). The loss happens due to Rayleigh scattering mechanism, where it is due to interaction of waves with a particle in the medium that removes energy from the directional propagation wave and transfers it to the other directions[6].

$$^{2}(P \quad \frac{1}{!\ _0} \frac{dP}{dt}) \quad \frac{1}{C^2} \frac{@^3 P}{@^2 t} = 0 \tag{8}$$

where $!\ _0$ is the damped resonance frequency and after doing a Fourier transform of the di erential equation he gets

$$\tilde{P}(t; R) = \int_{1}^{+1} P(t; R) e^{\ i!t}\ dt \tag{9}$$

$$\tilde{P}(!; R\ ) = \tilde{P}_0(!\ ) e^{\ (i \frac{!R}{C} + \frac{!\ ^2 R}{2!\ _0 c})} \tag{10}$$

Then he solves the wave equation neglecting the attenuation as done in the previous section, but instead of considering that the energy deposit is instantaneous he considered it as a Gaussian to get

$$\tilde{P}(!; R\ ) = i \frac{AE_0}{R} !e^{\ (i \frac{!R}{c} + \frac{^2 !\ ^2}{2} + \frac{!\ ^2 R}{2!\ _0 c})} \tag{11}$$

This whole work done before is in water where the attenuation coe cient is proportional

to $!^2$ and that is due to the movement of particles in the medium(Doppler shift).

Here comes Volovic who worked on solids and good enough for us, he worked in the moon regolith.

Volovic, in two of papers he participated in [7] [8], gave us the formula of pressure wave and the displacement wave.

$$\tilde{P}(!;R) = \frac{!}{r_s}\frac{E_0}{\ln(\frac{E_0}{E_c})}\sqrt{\frac{r_s}{R}}e^{\;!R} \tag{12}$$

where $=\frac{}{ct_0}$ with density of the material, Grunesien factor and t_0 is the radiation length

In solids the attenuation coe cient is proportional to $!$ and $=\frac{1}{2QC}$ where Q is the quality factor of the medium.

And the displacement wave equation

$$\frac{@^2u}{@t^2} \quad C^2 \quad u = \frac{F(r;t)}{} \tag{13}$$

and calculates the amplitude of displacement with respect to distance from the source.

$$u_{max}(R) = \frac{F}{}\frac{r_s^2}{C^2}\sqrt{\frac{r_s}{R}} \tag{14}$$

$u(r;t)$ u : displacement and : density of the medium

Where $F(r;t) = \frac{}{V}\frac{dE}{dx}$ is the bulk thermoelastic force, with V is the volume of the shower E is the energy deposited

2.5 Related Work

2.5.1 Niess and Bertin [2][3]

Oscillation Research with Cosmics in the Abyss(ORCA) is located at a depth around 2450m in the Mediterranean Sea 40 Km o Toulon's coast. It will be a neutrino telescope in the form of a water Cherenkov detector surrounded by 8 megatons of seawater. This detector is a 3d lattice of digital optical modules (DOMs) placed on 115 vertical strings, each holding 18 DOM.

A previous study was done by Valentin Niess and Vincent Bertin to detect ultra-high

energy neutrinos in sea water through acoustic waves that can be detected by hydrophones.

Figure 3: Sketch of the LSGA [4].

3 Methodology

After understanding the theory. The nal task reached is to see how the pressure wave evolves in the medium as it go further from the source.

As Learned, get the most clear solution between the three ones, since his pressure wave result comes with all its terms without taking its modulus as Askariyan or Volovic. But the problem that faces us here is that he is working for water case, so I take out the attenuation term and from replace it with that of Volovic (Solid attenuation). And since Learned is working with point like source (far eld as askariyan), while we need to work with surface waves (Rayleigh waves) because the shower of an ultra-high energy cosmic ray(proton) will happen on low depths (z 2m), I convert similar to Askariyan Learned equation to be as we need.

Final equation of pressure wave after Fourier transform of the wave equation given by Learned

$$\tilde{P}(!;R) = i\frac{AE_0}{R}!e^{\left(i\frac{!R}{c}+\frac{{}^2!\ {}^2}{2}+\frac{!\ {}^2R}{2!\ {}_0c}\right)} \tag{15}$$

here is the time needed by the particle to deposit its whole energy

$$= \frac{r_s}{C} \tag{16}$$

$$A = \frac{}{4\ C_p} \tag{17}$$

E_0 is the total energy deposit of the incoming particle

the last term of the exponential($\frac{!\ {}^2R}{2!\ {}_0c}$) represent the attenuation in a liquid medium.

Since we are working with moon rock(solid), replace the last term by the term from Volovic's paper $!R$ where ' $10^{\ 6}$[s=m],where he works in the case of the moon.

$$\tilde{P}(!;R) = i\frac{AE_0}{\sqrt{RR}}\sqrt{!}e^{\left(i\frac{!R}{c}+\frac{{}^2!\ {}^2}{2}+\ !R\ \right)} \tag{18}$$

Let's do an inverse Fourier transform of the equation.

For simplicity take

$$U(R) = \frac{AE_0}{\sqrt{RR}} \tag{19}$$

so

$$P(t;R) = \frac{U(R)}{2}\int_{1}^{+1} i!e^{\left(i\frac{!R}{c}+\frac{{}^2!\ {}^2}{2}+\ !R\ \right)}e^{i!t}\ d! \tag{20}$$

Here R is a constant, although $t = \frac{R}{c}$ as we know, but we are performing the Fourier transform at each speci c distance. So $t \neq \frac{R}{c}$

doing a small trick to simplify the integral

$$P(t;R) = \frac{U(R)}{2}\frac{d}{dt}\int_{-1}^{+1} e^{i\omega(t-\frac{R}{c})} e^{-(\frac{\sigma^2\omega^2}{2}+i\omega R)} d\omega \tag{21}$$

completing square of the second exponent

$$\frac{\sigma^2\omega^2}{2} + i\omega R = \frac{\sigma^2}{2}(\omega + \frac{R}{\sigma^2})^2 - \frac{1}{2}(\frac{R}{\sigma})^2 \tag{22}$$

$$P(t;R) = \frac{U(R)}{2}\frac{d}{dt}\int_{-1}^{+1} e^{i\omega(t-\frac{R}{c})} e^{-\frac{\sigma^2}{2}(\omega+\frac{R}{\sigma^2})^2} e^{\frac{1}{2}(\frac{R}{\sigma})^2} d\omega \tag{23}$$

take

$$t' = t - \frac{R}{c}) \quad dt' = dt \tag{24}$$

and

$$\omega' = \omega + \frac{R}{\sigma^2}) \quad d\omega' = d\omega \tag{25}$$

$$P(t',R) = \frac{U(R)}{2} e^{\frac{1}{2}(\frac{R}{\sigma})^2} \frac{d}{dt'}\int_{-1}^{+1} e^{i\omega' t'} e^{-\frac{\sigma^2}{2}\omega'^2} e^{-i\frac{R}{\sigma^2}t'} d\omega' \tag{26}$$

$$P(t',R) = \frac{U(R)}{2} e^{\frac{1}{2}(\frac{R}{\sigma})^2} \frac{d}{dt'} e^{-i\frac{R}{\sigma^2}t'} \int_{-1}^{+1} e^{i\omega' t'} e^{-\frac{\sigma^2}{2}\omega'^2} d\omega' \tag{27}$$

Complete square

$$i\omega' t' - \frac{\sigma^2}{2}\omega'^2 = -\frac{\sigma^2}{2}((\omega' - \frac{it'}{\sigma^2})^2 + \frac{t'^2}{\sigma^4}) \tag{28}$$

take

$$\omega'' = \omega' - \frac{it'}{\sigma^2}) \quad d\omega'' = d\omega' \tag{29}$$

$$P(t',R) = \frac{U(R)}{2} e^{\frac{1}{2}(\frac{R}{\sigma})^2} \frac{d}{dt'} e^{-i\frac{R}{\sigma^2}t'} e^{-\frac{t'^2}{2\sigma^2}} \int_{-1}^{+1} e^{-\frac{\sigma^2}{2}\omega''^2} d\omega'' \tag{30}$$

$$\int_{-1}^{+1} e^{-\frac{\sigma^2}{2}\omega''^2} d\omega'' = \frac{\sqrt{2}}{\sigma} \tag{31}$$

$$P(t',R) = \frac{U(R)}{2}\frac{\sqrt{2}}{\sigma} e^{\frac{1}{2}(\frac{R}{\sigma})^2} \frac{d}{dt'}(e^{-i\frac{R}{\sigma^2}t'} e^{-\frac{t'^2}{2\sigma^2}}) \tag{32}$$

$$P(t',R) = \frac{AE_0}{\sigma^3 R}\frac{\sqrt{2}}{\sigma}(-iR - t')e^{-i\frac{R}{\sigma^2}t' - \frac{t'^2}{2\sigma^2} + \frac{1}{2}(\frac{R}{\sigma})^2} \tag{33}$$

$$P(t^0, R) = \frac{AE_0}{2\pi^{3/2}R} \left(\quad i R \quad t^0 \right) e^{-\frac{1}{2}\frac{1}{\sigma^2}(t^0 + i R \quad)^2} \tag{34}$$

let, $t^{00} = i R \quad t^0)$ $\quad t^{00} = i R \quad + \frac{R}{c} \quad t$

$$P(t^{00}, R) = \frac{AE_0}{2\pi^{3/2}R} t^{00} e^{-\frac{t^{00 2}}{2\sigma^2}} \tag{35}$$

This result is very similar to that of Learned

3.1 Derive the dominant frequency

It is the frequency where the pressure wave peakes:

$$\tilde{P}(\omega; R) = i \frac{AE_0}{\sqrt{2\pi}R} \omega e^{\left(i \frac{\omega R}{c} + \frac{\sigma^2 \omega^2}{2} + \omega R \quad\right)} \tag{36}$$

$$\frac{d\tilde{P}}{d\omega}(\omega; R) = i \frac{AE_0}{\sqrt{2\pi}R} \omega \left(i \frac{R}{c} + \quad \sigma^2 \omega \quad + \omega R \quad\right) e^{\left(i \frac{\omega R}{c} + \frac{\sigma^2 \omega^2}{2} + \omega R \quad\right)} = 0 \tag{37}$$

$$\omega \left(i \frac{R}{c} + \quad \sigma^2 \omega \quad + \omega R \quad\right) = 0 \tag{38}$$

3.2 Pressure of instant energy deposition

Here I take the previous equation without the σ^2 factor, and since we are only interested in the positive frequencies with $|\omega| < \omega_{max}$

$$\tilde{P}(|\omega|; R) = i \frac{AE_0}{\sqrt{2\pi}R} |\omega| e^{\left(i \frac{|\omega| R}{c} + \quad |\omega| R\right)} \tag{39}$$

$$P(t; R) = \frac{1}{2} \int_{-\omega_{max}}^{\omega_{max}} \tilde{P}(|\omega|; R) e^{i|\omega| t} d|\omega| \tag{40}$$

We have an even integral

$$P(t^0, R) = 2 Z(R) \int_0^{\omega_{max}} \omega e^{k(t^0; R)\omega} d\omega \tag{41}$$

where $Z(R) = \frac{iAE_0}{2R}$, $t^0 = t \quad \frac{R}{c}$ and $k(t^0; R) = it^0 \quad R$

Integration by parts

$$P(t^0; R) = 2Z(R)[\frac{!e^{k(t^0;R)!}}{k(t^0; R)} j_0^{!_{max}} \quad \frac{1}{k(t^0; R)} \int_0^{!_{max}} e^{k(t^0;R)!} d! \,] \tag{42}$$

$$P(t^0; R) = 2Z(R)[\frac{!_{max} e^{k(t^0;R)!_{max}}}{k(t^0; R)} \quad \frac{1}{k(t^0; R)^2}(e^{k(t^0;R)!_{max}} \quad 1)] \tag{43}$$

$$P(t^0; R) = \frac{2Z(R)}{k(t^0; R)}[e^{k(t^0;R)!_{max}} (!_{max} \quad \frac{1}{k(t^0; R)^2})e^{k(t^0;R)!_{max}} \quad 1] \tag{44}$$

3.3 Askariyan and Volovic Comparison

It is somewhat di cult which model to use for our work, but rst let's be sure that the three of them is compatible with each other.

Askariyan solves the pressure wave equations in the near and the far eld cases.

Volovik works with the near eld case, if we compare it with the far eld of Askariyan it is the same to the factor of disregarding the attenuation and the Gaussian distribution of the energy deposit.

Askariyan's equation in the near eld is as follows

$$\tilde{P}(!;R) = \frac{!a}{4\ ^{\frac{3}{2}}C_p}\,p\,\frac{E}{\sqrt{RR}} \tag{45}$$

Where

$R = \frac{L^2}{}$ where L is the length of the shower

$f_{cr} = \frac{c}{2}$ is the Shannon frequency, where $f_{cr} = \frac{f_{obs}}{2}$) $f_{obs} = \frac{c}{r_s}$

$$\tilde{P}(!;R) = \frac{!aE}{4\ ^{2}C_pL}\sqrt{\frac{r_s}{R}} \tag{46}$$

This equation we get is not taking into consideration the attenuation of the medium.

Volovic gives this equation

$$\tilde{P}(!;R) = \frac{!}{r_s}\frac{E_0}{\ln(\frac{E_0}{E_c})}\sqrt{\frac{r_s}{R}}e^{\ !R} \tag{47}$$

Taking out the attenuation factor and the $\ln(\frac{E_0}{E_c})$ we get

$$\tilde{P}(!;R) = \frac{!E_0}{r_s}\sqrt{\frac{r_s}{R}} \tag{48}$$

Some assumptions done to compare Askariyan with Volovic

$\overline{c} = \frac{c}{C_p}$, $\overline{t_0} = L^{\ 1}$

so $= \frac{C}{C_pL}$

they match dimensionally

$$\tilde{P}(!;R) = \frac{c}{r_s}\frac{!aE_0}{C_pL}\sqrt{\frac{r_s}{R}} \tag{49}$$

Volovic also mentions! $_{max} = \frac{c}{r_s}$

! $_{max}$ f $_{obs}$

We can recognize that Askariyan gives the pressure per frequency and as he mentioned in his paper.

$$\tilde{P}_{Askariyan}(!; R) = \frac{1}{2}\frac{\tilde{P}_{V\,olovic}(!; R)}{2\,f_{obs}} \tag{50}$$

We can see that Volovic and Askariyan di er by a factor of 2.

Most important is that Volovic's pressure wave is multiplied by an extra! because we will do an inverse Fourier transform that will add a [Hz] because it is integrated over!

3.4 Displacement and Pressure wave

It is also possible to work on displacement of the medium and see how it evolves and how it transmits to the ber.

Let's see if the displacement di erential equation is the same as that of pressure in Volovic's literature.

Figure 4: Sketch to how displacement wave propagates in a medium [9]

$V_i; V_f$: initial, nal volume before and after the wave comes

: volumetric modulus of elasticity

$V_i = A(x_2 \quad x_1); V_f = A(x_2 + y_2 \quad x_1 \quad y_1)$

$V = V_f \quad V_i = A(y_2 \quad y_1)$

$$P = \frac{V}{V} = \frac{y_2 \quad y_1}{x_2 \quad x_1} = \frac{dy}{dx} \tag{51}$$

f: net force ; A: base area of the cylinder

P_1 and P_2 are the pressures at each side of the cylinder

$$f = A(P_1 \quad P_2) \tag{52}$$

$m = V_i = A(x_2 \quad x_1)$

According to Newton's law

$$F = \frac{d^2y}{dt^2} \tag{53}$$

$$) \quad \frac{dP}{dx} = \quad \frac{d^2y}{dt^2} \tag{54}$$

Volovic wave equation of displacement:

$$\frac{@^2u}{@t^2} \quad C^2 \quad u = \frac{F(r;t)}{} \tag{55}$$

$$\frac{@^2u}{@t^2} \quad C^2 \quad u = -\frac{dq}{dx} \tag{56}$$

$$\frac{dP}{dx} = \quad \frac{d^2y}{dt^2} \tag{57}$$

$\frac{@}{@t}$ and commute with themselves and with each other

we can integrate twice this equation by time and derive once by x to get:

$$\frac{@^2P}{@t^2} \quad C^2 \quad P = \quad \frac{@^2q}{@t^2} \tag{58}$$

which is the pressure di erential equation of Askariyan

So the two di erential equations represent the same thing

3.5 Numerical approach

The approach that I did at rst was use some code to get some results. Here a python code is developed to insert the values of pressure then do a discrete Fourier transform and then nds the amplitude of the pressure wave at each distance.

3.5.1 Values needed

To perform a simulation of the pressure wave it is necessary to know some constants in the moon regolith such as the speed of sound, thermal expansion coe cient, heat capacity, quality factor and the radius and length of the shower. The last value is estimated for the lack of a proper simulation of the shower($_s$ = 0:1mandL = 1m).

C = 100-300 m/s (we will take the worse scenario C = 100) Q = 4000-8000 (take Q 5000

As we know = $\frac{1}{2QC}$ = 10 ^{6}s=m

	T = 90k	T = 350k
C_p 10^7[Jkg ^{1}k ^{1}]	8	80
10 ^{5}[k ^{1}]	1	2
=C^2	0.03	0.006

Dominant frequency is at where the pressure wave peaks. At each distance there is a dominant frequency, I get it by nding the index of maximum pressure value and then nding it in the frequency array.

3.5.2 DFT and the maximum Pressure

I did a function that performs discrete inverse Fourier transforms instead of using i t function to be able to mention negative time.

Lastly, we do a function that nds the maximum pressure at each distance. We combine

the previous functions.

We create the array of maximum pressure and in two for loops where we use the information of the dominant frequency to construct the frequency axis at each distance, then we do the same thing as in the rst function to calculate the pressure wave in the frequency domain and ll it in an array then apply inverse Fourier transforms on it. After taking the absolute of the array, take the maximum element and enter it in the array of maximum pressure.